

compliance constants s_{ij} determined many years ago by Bridgman.² The s_{ij} determined by Bridgman are isothermal values; the c_{ij} values reported here are adiabatic. We have, therefore, converted Bridgman's values of s_{ij} to isothermal c_{ij} by inverting the matrix of Eq. (1), and thence to adiabatic c_{ij} , by use of thermodynamic formulas of the standard type.²¹ These values are also given in Table II. Finally, we have inverted our c_{ij} matrix to obtain adiabatic s_{ij} which within experimental error are the same as the isothermal values. It is to be noted that our values of the c_{ij} obtained by inverting Bridgman's s_{ij} are materially different from those reported by Betts *et al.*²²

We focus attention on the comparison of our adiabatic measured values of c_{ij} with the adiabatic values of the c_{ij} computed from Bridgman's static s_{ij} . Several large discrepancies, far larger than the combined estimated experimental errors, are apparent for c_{33} and c_{12} . The origin of these discrepancies is difficult to ascertain. However, we note that Bridgman's values are only partially redundant, that several of his samples were reported as polycrystalline, and that the purity of his samples was dubious. We suspect a gross orientational error²³ (perhaps concealed as an occluded grain or an unnoticed twin) in one or more of his crystals. The discrepancy in the sign of c_{14} is only apparent and arises presumably because of a difference between Bridgman's unspecified axis convention and that used here.

A more rewarding comparison (see Table IV) is afforded by the agreement between Bridgman's directly observed adiabatic volume compressibility and that calculated from our data: the former is 29.3×10^{-13} cm²/d and the latter, 30.7×10^{-13} cm²/d. The difference between these two values is well within the combined experimental error.

We turn now to the question of whether the Voigt theory provides an adequate description of the elastic properties of bismuth. In view of the numerous redundant checks, the answer must certainly be affirmative to within the accuracy of our experiments. However, in view of the existing uncertainty over the applicability of the Laval-Raman theory to various crystals, it might be worthwhile to set some quantitative limits on the permitted deviations from the Voigt framework, at least, as applied to bismuth.

Briefly, the difference between the Voigt and the Laval-Raman theory may be stated as follows. The Voigt theory considers the case of a static (or homogeneous) strain in which the strain tensor may be separated into symmetric and antisymmetric components. The latter correspond to rigid body rotations which invoke no stress. The stress tensor may then be proved symmetric. In a dynamic and, consequently, inhomogeneous strain, the rotational part of the displacement has a spatial variation and hence requires torques to sustain the changes in angular momentum. It is therefore impossible to demonstrate that the stress tensor is symmetric. In the most general case, one requires 45 constants to relate the stress to the strain, after imposing exactness conditions, rather than 21 constants as in the Voigt theory. Wooster has shown that this number is further reduced by compatibility requirements to 39 constants. This number of constants is further reduced by the requirements of crystal symmetry so that for the simplest crystal classes (e.g., isotropic materials), there is no difference between the two theories.

Particularizing to the case of Bi, one finds that eight constants are required to specify the relation between stress components X_{ij} and strain components x_{ij} . In fact,^{24,25}

$$X_{ij} = \begin{vmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & 0 & 0 & 0 \\ & d_{11} & -d_{13} & -d_{14} & -d_{15} & 0 & 0 & 0 \\ & & d_{33} & 0 & 0 & 0 & 0 & 0 \\ & & & d_{44} & \frac{d_{44}+d_{55}}{2} & 0 & 0 & 0 \\ & & & & d_{55} & \frac{d_{44}+d_{55}}{2} & d_{15} & d_{15} \\ & & & & & d_{55} & d_{14} & d_{14} \\ & & & & & & d_{66} & d_{66} \\ & & & & & & & d_{66} \end{vmatrix} x_{ij} \quad (16)$$

²¹ See J. F. Nye, *Physical Properties of Crystals* (Oxford University Press, Oxford, England, 1957), p. 283.

²² D. D. Betts, A. B. Bhatnagar, and G. K. Horton, *Phys. Rev.* **104**, 43 (1956).

²³ The discrepancy between the value for V_{12} previously reported by one of us and that in Table II was owing to a "false" trigonal axis being mislabeled in a Laue pattern.

²⁴ Note that this matrix differs slightly from that given by Raman and Viswanathan owing to a misprint in their article. We have used their notation which differs from that of LeCorre.

²⁵ Y. LeCorre, *Bull. Soc. franc. Mines. Crist.* **70**, 80 (1957).